VR Project 2015 – 2018

IDENTIFICATION OF CONSTITUTIVE PARAMETERS IN ARTERIES

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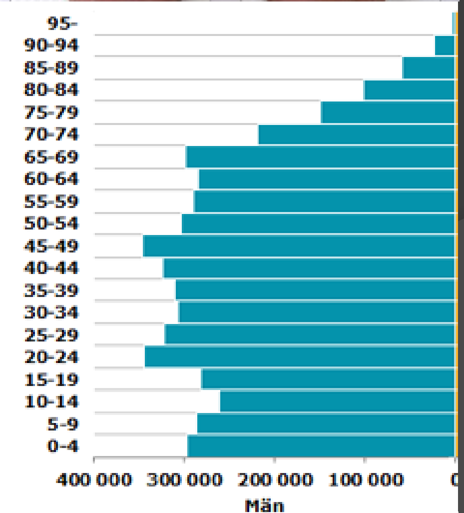
Motivation

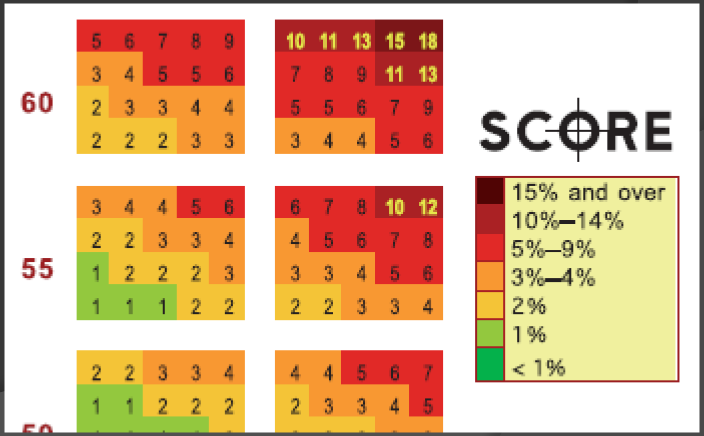
- Cardiovascular (CV) diseases cause 40% of all deaths in the EU
- The total cost for CV diseases is 196 billion Euro
- Age, cholesterol, diabetes, hypertension, obesity, and smoking are risk markers



The Swedish demography (www.scb.se)

Motivation (cont'd.)

- Prediction of CV events, e.g., SCORE
- Underestimation of risk for single marker subjects
- Fluctuations in markers over time

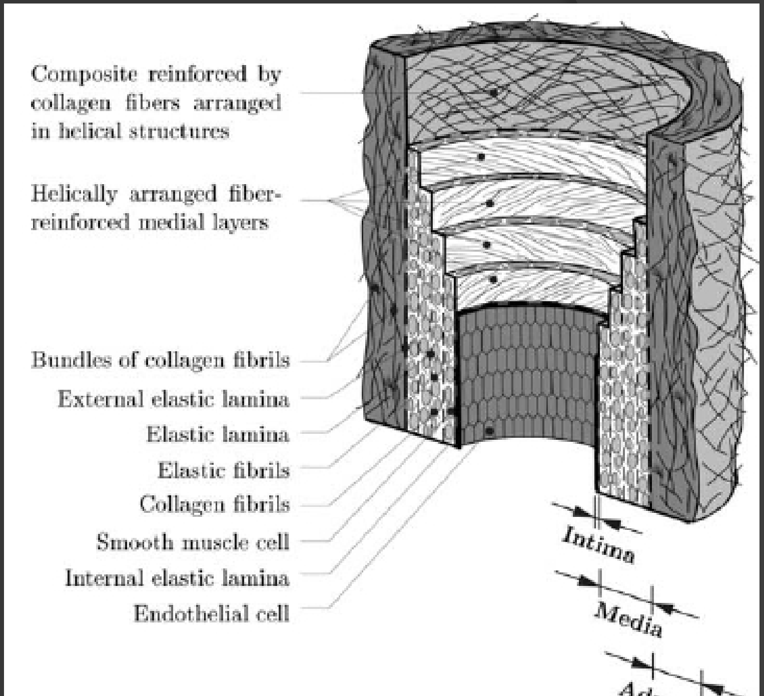


Motivation (cont'd.)

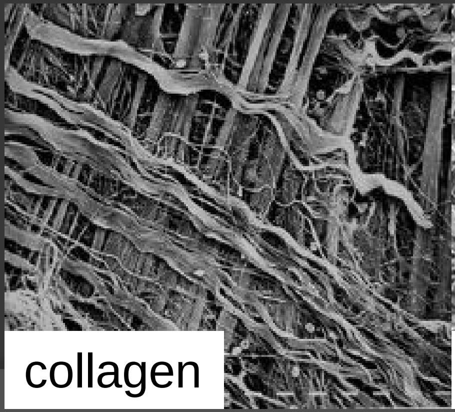
- Arterial stiffness is a potential risk marker
- Increased arterial stiffness is an essential ingredient to develop hypertension
- Reflects the 'cumulative risk'

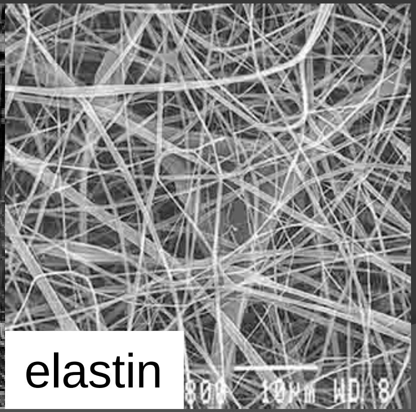
Physiology

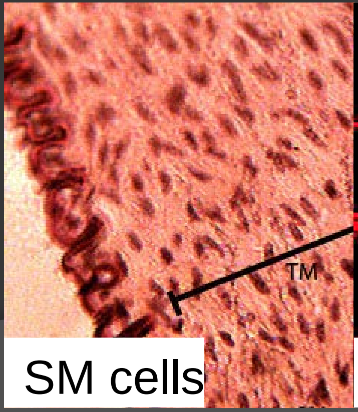
- Extracellular matrix of collagen and elastin
- Circumferentially embedded smooth muscle (SM) cells
- Increases focal adhesion, intrinsic SM stiffness

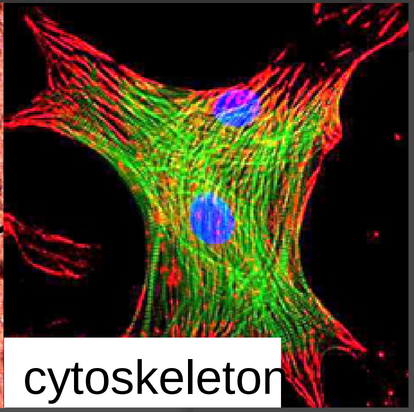


Holzapfel et al, J Elast, 61, 2000



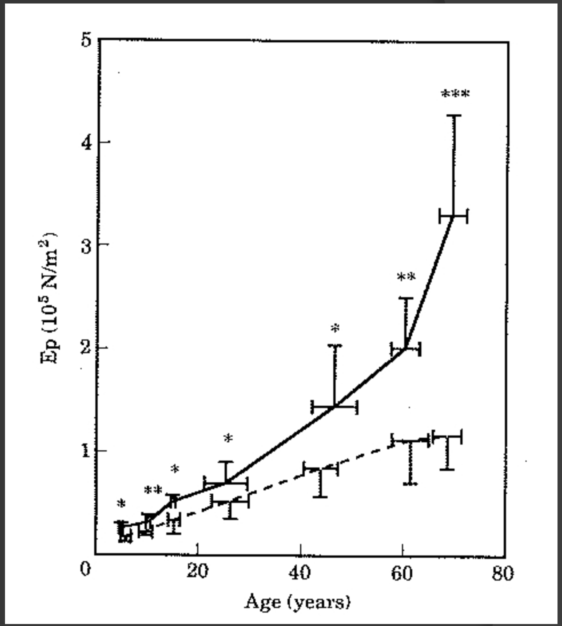






Arterial Stiffness Estimation

- Clinical measures are crude
- Constitutive parameters from nonlinear models: van der Horst et al. (2012), Masson et al. (2008), Schulze-Bauer & Holzapfel (2003), Smoljkic et al. (in press), Stalhand and Klarbring (2005), Stalhand (2009), Astrand et al. (2011)
- Trade-off between details and identifiability and more complex



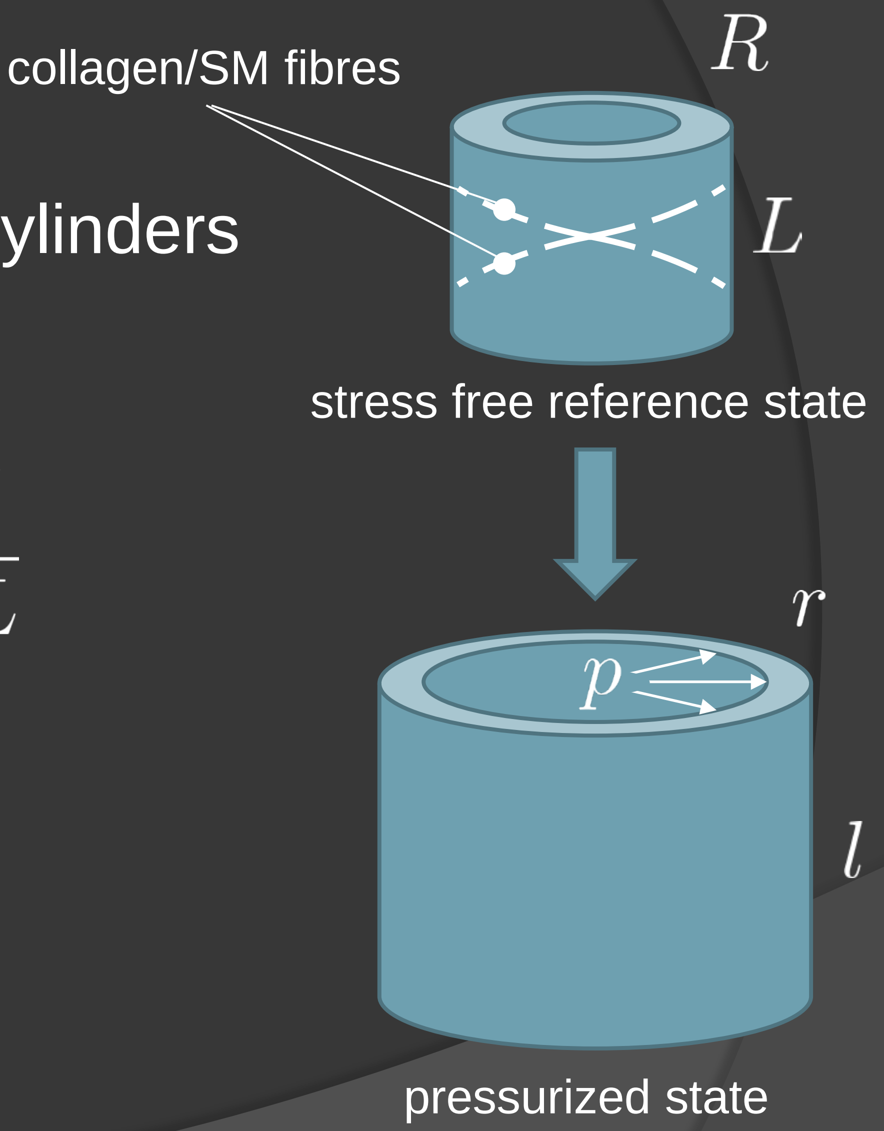
Kinematics

- Pressurized thin-walled cylinders
- Shear free inflation

$$\lambda_\theta = \frac{r}{R}, \qquad \lambda_z = \frac{l}{L}$$

- Incompressibility

$$\lambda_\theta \lambda_z \lambda_r = 1$$



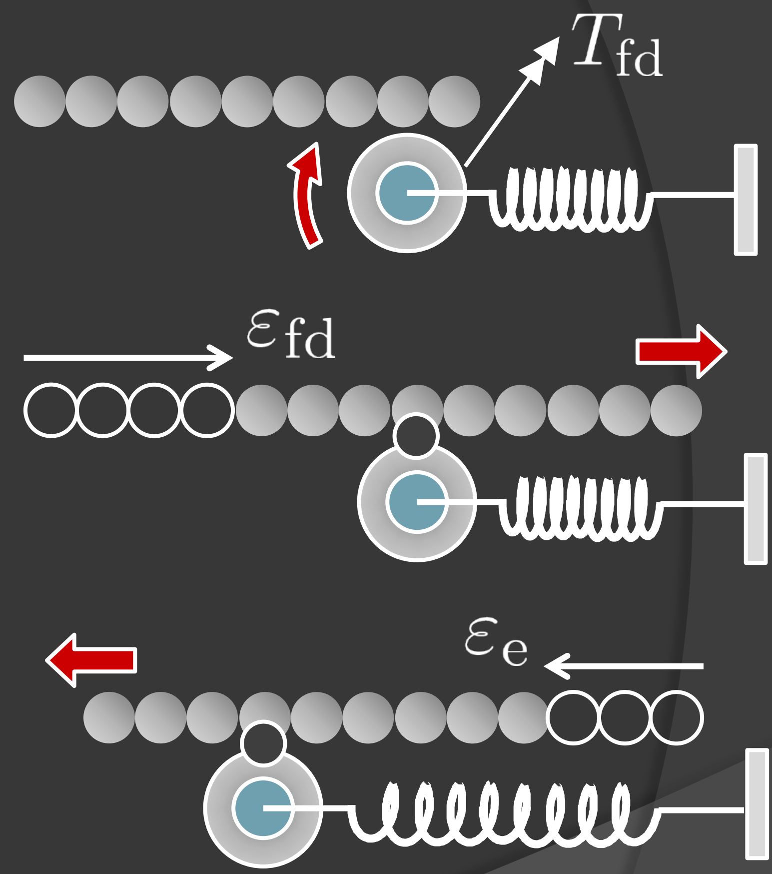
Kinematics (cont'd.)

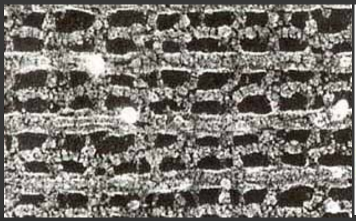
- Active force generation by a spring-friction-clutch model
- Tangential smooth muscle arrangement

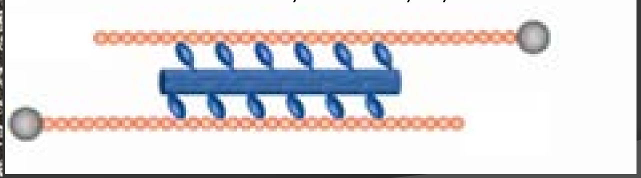
$$\lambda_\theta = \varepsilon_{\mathrm{e}} + \varepsilon_{\mathrm{fd}} + 1$$

elastic deformation

filament translation







Murtada et al, BMMB, 9, 2010

Constitutive Equations

- Dissipation inequality:

$$T_\theta \dot{\lambda}_\theta + T_\mathrm{z} \dot{\lambda}_\mathrm{z} + T_\mathrm{r} \dot{\lambda}_\mathrm{r} + T_\mathrm{fd} \dot{\varepsilon}_\mathrm{fd} - \dot{\psi} \geq 0$$

- Free energy:

$$\psi = \psi^\mathrm{p} + \psi^\mathrm{a} - p\left(\lambda_\theta \lambda_\mathrm{z} \lambda_\mathrm{r} - 1\right),$$

$$\psi^\mathrm{p} = \psi^\mathrm{p}(\lambda_\theta, \lambda_\mathrm{z}, \lambda_\mathrm{r}), \quad \psi^\mathrm{a} = \psi^\mathrm{a}(\varepsilon_\mathrm{e}, \varepsilon_\mathrm{fd})$$

Constitutive Equations (cont'd.)

$$T_\theta = -\frac{p}{\lambda_\theta} + \frac{\partial \psi^{\mathrm{p}}}{\partial \lambda_\theta} + \frac{\partial \psi^{\mathrm{a}}}{\partial \varepsilon_{\mathrm{e}}} \qquad \text{(first P-K stress)}$$

$$T_{\mathrm{z}} = -\frac{p}{\lambda_{\mathrm{z}}} + \frac{\partial \psi^{\mathrm{p}}}{\partial \lambda_{\mathrm{z}}}$$

$$T_{\mathrm{r}} = -\frac{p}{\lambda_{\mathrm{r}}} + \frac{\partial \psi^{\mathrm{p}}}{\partial \lambda_{\mathrm{r}}}$$

$$T_{\mathrm{fd}} - \frac{\partial \psi^{\mathrm{a}}}{\partial \varepsilon_{\mathrm{fd}}} + \frac{\partial \psi^{\mathrm{a}}}{\partial \varepsilon_{\mathrm{e}}} = c\dot{\varepsilon}_{\mathrm{fd}} \qquad \text{(evolution law)}$$

$$\sigma_k = \lambda_k T_k, \qquad \text{(Cauchy stress)}$$

Free-energy functions

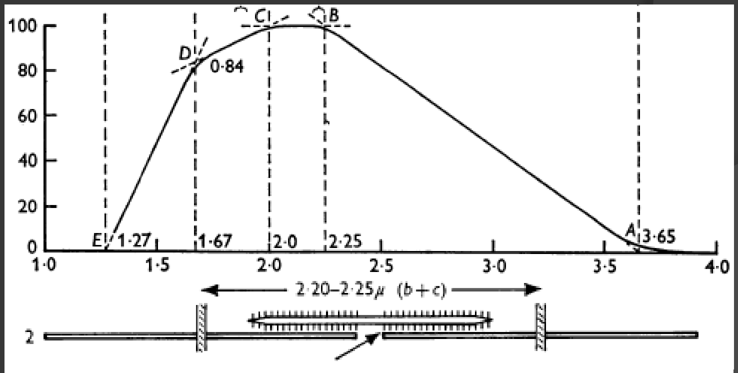
- Passive tissue (tangential collagen):

$$\psi^{\mathrm{p}} = \frac{c_0}{2}\left(\lambda_\theta^2 + \lambda_{\mathrm{z}}^2 + \lambda_{\mathrm{r}}^2 - 3\right) + \frac{c_1}{c_2}\left(\exp\{c_2(\lambda_\theta^2 - 1)^2\} - 1\right)$$

- Actomyosin network (linear strain energy):

$$\psi^{\mathrm{a}} = \frac{1}{2} f K_{\mathrm{e}} \varepsilon_{\mathrm{e}}^2$$

$$f = \exp\{\left(\varepsilon_{\mathrm{a}}^0 - \varepsilon_{\mathrm{a}}\right)^2 / 2\gamma^2\}$$



Gordon et al, JAP, 184, 1966

Specialized Evolution Law

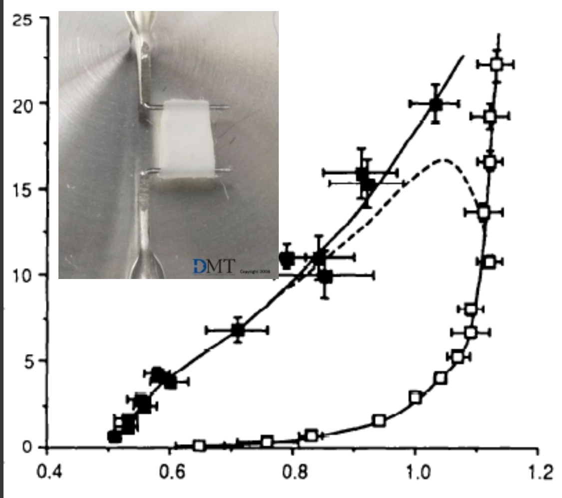
- Quasi-static conditions: $\dot{\varepsilon}_{\mathrm{fd}} = 0$
- Non-cycling (latch) cross-bridges: $T_{\mathrm{fd}} = 0$
- Specialized evolution law:

$$f\varepsilon_{\mathrm{e}} - \frac{1}{2}\frac{\partial f}{\partial \varepsilon_{\mathrm{fd}}}\varepsilon_{\mathrm{e}}^{2} = 0$$

Parameter Estimation

- Wire myograph
  - 1D
- Pressure-radius response
  - Incomplete data
  - Reference geometry?
  - Multiple minima

Passive			Active			Geom	
c_0	c_1	c_2	K_e	ε_{fd}^0	γ	L	R



Davids and Gore, J Physiol, 256, 1989